Variations of the Interval Linear Assignment Problems

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Abstract: - In this paper deals with the Assignment problems arise in different situation where we have to find an optimal way to assign n objects to m other objects. These problems to find numeral application in production planning, Sales proportion, air –line operators etc. For example using maximizes (or) minimizes assignment methods and the existing Hungarian methods have been solved. In the entries of the cost matrix is not always crisp. In many application this parameters are uncertain and this uncertain parameters are represented by interval. In this contribution we propose interval Hungarian method and consider interval analysis concept for solving interval linear assignment problems.

*Keywords:-*Assignment Problems, Hungarian method, optimum solution, Interval Linear Assignment Problems, Maxima and Minima, Alternative optimal solution

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I. INTRODUCTION

An assignment problem is a particular case of transportation problem where the objective is to assign a number of resources to an equal number of activities on a one to one basis so as to minimize total cost or maximize total profit of allocation. The problem of assignment arises because available resources such as men, machines, production etc. Thus, the problem is how the assignments should be made so as to optimize the given objective. The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operations research in Mathematics.

It is worth to recall that the assignment problem has been used in a variety of application contexts such as personnel scheduling, manpower planning and resource allocation. The standard assignment problem can be seen as a relaxation of more complex combinatorial optimization problems such as traveling salesman problem quadratic assignment problem etc. It can also be considered as a particular transportation problem with all supplies and demands equal to 1. The assignment problem has also several variations such as the semi-assignment problem and the k-cardinality assignment problem. The reader interested in more details about these two problems or other variations for a comprehensive survey of the assignment problem variations.

II. **DEFINITION**

Arithmetic Operations in interval The interval form of the parameters may be written as where is the left value [\underline{x}] and is the right value [\overline{x}] of the

interval respectively. We define the centre is $m = \frac{\overline{x} + \underline{x}}{2}$ and $w = \overline{x} - \underline{x}$ is the width of the interval $[\overline{x}, \underline{x}]$ Let $[\overline{x}, \underline{x}]$ and $[\overline{y}, \overline{y}]$ be two elements then the following arithmetic are well known

(i) $[\overline{x}, \underline{x}] + [\overline{y}, y] = [\overline{x} + \overline{y}, \underline{x} + y]$

(ii) $[\overline{x}, \underline{x}] \times [\overline{y}, y] = [\min\{\overline{x}y \ \underline{x}\overline{y}, \overline{xy}, \underline{x}y\}, \max\{\overline{x}y \ \underline{x}\overline{y}, \overline{xy}, \underline{x}y\}]$

(iii) $[\overline{x}, \underline{x}] \div [\overline{y}, \underline{y}] = [\min\{\overline{x} \div \underline{y} \ \underline{x} \div \overline{y}, \overline{x} \div \overline{y}, \underline{x} \div \underline{y}\}, \max\{\overline{x} \div \underline{y} \ \underline{x} \div \overline{y}, \overline{x} \div \overline{y}, \underline{x} \div \underline{y}\}]$ provide if $[\overline{y}, y] \neq [0, 0],$

(iv)
$$[\overline{x}, \underline{x}] - [\overline{y}, \underline{y}] = [\overline{x} - \underline{y}, \underline{x} - \overline{y}]$$
 Provide if $[\overline{y}, \underline{y}] \neq [0, 0]$,

III. MATHEMATICAL MODEL OF ASSIGNMENT PROBLEM

Given n resources (or facilities) and activities (or jobs), and effectiveness (in terms of cost, profit, time, etc) of each resource (facility) for each activity (job), the problem lies in assigning each resource to one and only one activity (job) so that the given measure of effectiveness is optimized.

| Resources | | | Supply |
|----------------|-----------------|----------------------|--------|
| (Workers) | J1 | J2,J3Jn | 1 |
| W_1 | C ₁₁ | $C_{12}C_{13}C_{1n}$ | 1 |
| W_2 | C ₂₁ | $C_{22}C_{23}C_{2n}$ | 1 |
| | | | |
| | | | |
| W _n | C _{n1} | $C_{n2}C_{n3}C_{nn}$ | 1 |
| Demand | 1 | 1,1,11 | n |

From the table, it may be noted that the data matrix is the same as the transportation cost matrix expect that supply (or availability) of each of the resources and the demand at each of the destinations is taken to be one .it is due to this fact that assignments are made on a one-to-one basis

Let X_{ij} denote the assignment of facility I to job j such that

$$Xij = \begin{cases} 1 & if facility is assigned to job, j \\ 0 & otherwise \end{cases}$$

Then, the mathematical formulation of the standard assignment problem (SAP) is as follows:

Min Z= $\sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$ Subject to

$$\sum_{i=1}^{1} x_{ii} = 1$$
 i=1,2-----n

$$\sum_{i=1}^{1} x_{ij} = 1 \quad j=1,2 - \dots - n$$

 $x_{ij} = \{0 \text{ or } 1\}$ i, j = 1, 2-----n

where for all i, j = 1, ..., n, cij is the cost of assigning agent I to task j, Xij = 1 means that agent i is assigned to task j and Xij = 0 means that agent i is not assigned to task j.The first set of constraints implies that each agent is assigned to one and only one task and the second set of constraints implies that to each task is assigned one and only one agent.

In addition to the minimization of assignment cost, an assignment problem may consider other objective functions such as the minimization of completion time. When the assignment problem is considered with the minimization of assignment cost as the objective function, it is called the cost minimizing assignment problem.

Note:-

It may be noted that assignment problem is a variation of transportation problem with two characteristics 1. The cost matrix is a square matrix 2. The optimum solution for the problem would always be such that there would be only one assignment in a given row or column of the cost matrix

IV. MANAGERIAL APPLICATIONS OF THE ASSIGNMENT METHOD:



1. Natural applications

- This field is applicable to:
- 1. Match jobs to machines.
- 2. Assign sales people to sales territories.
- 3. Assign accountants to client accounts.
- 4. Assign contracts to bidders through systematic evaluation of bids from competing suppliers.
- 5. Assign naval vessels to petrol sectors.
- 6. Assign development engineers to several construction sites.
- 7. Schedule teachers to classes etc.

8. Men are matched to machines according to pieces produced per hour by each individual on each machine.

9. Teams are matched to project by the expected cost of each team to accomplish each project.

2.Non-obvious applications

- 1. Vehicles routing.
- 2. Signal processing
- 3. Virtual output queueing
- 4. Multiple object tracking
- 5. Approximate string matching

V. VARIATIONS OF THE ASSIGNMENT PROBLEM



1. Non-square matrix (Unbalanced assignment problem):

Such a problem is found to exist when the number of facilities is not equal to the number of jobs. Since the Hungarian method of solution requires a square matrix, fictious facilities or jobs may be added and zero costs be assigned to the corresponding cells of the matrix. These cells are then treated the same way as the real cost cells during the solution procedure.

2. Maxima and Minima method:

Sometimes the assignment problem may deal with maximization of the objective function. The maximization problem has to be changed to minimization before the Hungarian method may be applied. This transformation may be done in either of the following two ways:

- a. by subtracting all the elements from the largest element of the matrix.
- b. by multiplying the matrix elements by-1

3. In feasible assignment problem (constrained):

A constrained assignment occurs in the cell (i, j) of the assignment cost matrix if i^{th} person is unable to perform j th job. Such problems can be solved by assigning a very heavy cost (infinite cost) to the corresponding cell. Such a job will then be automatically excluded from further consideration.

In such cases, the cost of performing that particular activity by a particular resource is considered to be very large (written as M or ∞) so as to prohibit the entry of this pair of resources- activity into the final solution.

4. Alternate optimal solution:

Sometimes, it is possible to have two or more ways to strike off all zero elements in the reduce matrix for a given problem. In such cases there will be alternate optimal solutions with the same cost. Alternate optimal solutions offer a great flexibility to the management sine it can select the one which is most suitable to its requirement.

6. Solution Methods of Assignment Problem:



Example 1(Maximization problem):

A company has four territories open and four salesmen available for assignment. The territories are not equally rich in their sales potential. It is estimated that a typical salesman operation in each territory would bring in the following annual sales:

 Territory:
 I
 II
 III
 IV

 Annual sales (RS) : 60,000
 50,000
 40,000
 30,000

The four salesmen are also considered to differ in ability: it is estimated that working under the same conditions, their yearly sales would be proportionately as follows:

Salesman:ABCDProportion:7554

If the criterion is maximum expected total sales, the intuitive answer is to assign the best salesman to the richest territory; the next best salesmen to the second richest territory and so on verify this answer by the assignment method

Solution:

Step 1: To construct the effectiveness of the matrix .By taking Rs. 10000/- as one unit and the sales proportion and the maximum sales matrix is obtained as follows:

| Table (a) | | | | | | | | |
|--------------------------------|---------|----------|---------------|------------|-------|----|--|--|
| Sales in 10 thousand of rupees | | | | | | | | |
| Sales Proportio | n | | | - | - | | | |
| | | | 6 | 5 | 4 | 3 | | |
| | | | Ι | II | III | IV | | |
| | 7 | А | 42 | 35 | 28 | 21 | | |
| | 5 | В | 30 | 25 | 20 | 15 | | |
| | 5 | С | 30 | 25 | 20 | 15 | | |
| | 4 | D | 24 | 20 | 16 | 12 | | |
| the value of C | - Sales | Proporti | on \times S | Sales Terr | itory | | | |

 $\mathbf{T}_{a}\mathbf{h}\mathbf{1}_{a}$ (a)

Find the value of C_{11} = Sales Proportion × Sales Ter 7 ×6 = 42

In the same it is continued for the remaining cells **Step 2:**

We take a linear assignment problem as an example problem and solved this problem by traditional Hungarian method. The assignment cost of assigning any operator to any one machine is given in the following table Table (b)

Cost matrix with crisp entries

| | Ι | Π | III | IV |
|---|---------|---------|---------|---------|
| A | [41,43] | [34,36] | [27,29] | [20,22] |
| В | [29,31] | [24,26] | [19,21] | [14,16] |
| С | [29,31] | [24,26] | [19,21] | [14,16] |
| D | [23,25] | [19,21] | [15,17] | [11,13] |

Step 3:

To convert the maximum sales matrix to minimum sales matrix .By simply multiplying each element of given matrix by -1. Thus resulting matrix becomes:

Table (C)

Cost matrix with crisp entries

| | Ι | II | III | IV |
|---|-----------|-----------|-----------|-----------|
| А | [-41,-43] | [-34,-36] | [-27,-29] | [-20,-22] |
| В | [-29,-31] | [-24,-26] | [-19,-21] | [-14,-16] |
| С | [-29,-31] | [-24,-26] | [-19,-21] | [-14,-16] |
| D | [-23,-25] | [-19,-21] | [-15,-17] | [-11,-13] |

Now, using the above table we can apply the Hungarian method to find the assignment for the given problem and the value should be taken from the original table since, it is a maximization problem

Step 4: Select the most negative in the matrix (i.e) is [-41,-43]. With this element subtract all the Elements in the matrix. MinZ = -(-MaxZ), the resulting is minimization table

Table (C)

| | Ι | II | III | IV |
|---|---------|---------|---------|---------|
| А | [0,0] | [7,7] | [14,14] | [21,21] |
| В | [12,12] | [17,17] | [22,22] | [27,27] |
| С | [12,12] | [17,17] | [22,22] | [27,27] |
| D | [18,18] | [22,22] | [26,26] | [30,30] |

Find the value of $C_{11} = [-41, -43] - [-41, -43] = [-41+41, -43+43] = [0, 0]$ in the same it is continued for the remaining cells

Step 5. Iterate towards an Optimal Solution. We proceed according to the Hungarian algorithm and we get optimal solution

| [0, | 0] | [2, | 2] | [4,4 | -] | [7,7 |] |
|-----|----|------|----|------|----|------|---|
| [0, | [0 | [0, | 0] | Ð | ¥ | [1,1 |] |
| [0, | [0 | [0,0 | [] | [0, |)] | [1,1 |] |
| [2, | 2] | [1, |]] | [0, |)] | [0,0 |] |
| | | | | | | | |
| | | | | | | | |

We are applying the proposed interval Hungarian method and solve this problem. We get an maximum assignment cost is [95, 103] and optimal assignment as A,B,C,D machines are assign to I,II, III, IV operators respectively.

Example 2:

Beta Corporation has four plants each of which can manufacture any one of four products Production costs differ from one plant to another as do sales revenue. Given the revenue and cost data below, obtain which product each plant should produce to maximize profit.

Plant

Sales revenue (Rs. 000s Product)

| | 1 | 2 | 3 | 4 |
|---|----|----|----|----|
| А | 50 | 68 | 49 | 62 |
| В | 60 | 70 | 51 | 74 |
| С | 55 | 67 | 53 | 70 |
| D | 58 | 65 | 54 | 69 |

Production costs (Rs. 000s Product)

Plant

| | 1 | 2 | 3 | 4 |
|---|----|----|----|----|
| А | 49 | 60 | 45 | 61 |
| В | 55 | 63 | 45 | 69 |
| С | 52 | 62 | 49 | 68 |
| D | 55 | 64 | 48 | 66 |

Solution:

Step 1: Now, we have found the profit matrix by using sales revenue and production cost. Profit = sales - cost

Profit matrix

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| А | 1 | 8 | 4 | 1 |
| В | 5 | 7 | 6 | 5 |
| С | 3 | 5 | 4 | 2 |
| D | 3 | 1 | 6 | 3 |

Step2:

We take a linear assignment problem as an example problem and solved this problem by traditional Hungarian method .The assignment cost of assigning any operator to any one machine is given in the following table Table (a)

| Cost matrix with crisp entries | | | | | | | |
|--------------------------------|-------|-------|-------|-------|--|--|--|
| - | Ι | II | III | IV | | | |
| А | [0,2] | [7,9] | [3,5] | [0,2] | | | |
| В | [4,6] | [6,8] | [5,7] | [4,6] | | | |
| С | [2,4] | [4,6] | [3,5] | [1,3] | | | |
| D | [2,4] | [0,2] | [5,7] | [2,4] | | | |

Step3: To convert the maximum sales matrix to minimum sales matrix.By simply multiplying each element of given matrix by -1. Thus resulting matrix becomes:

| | ruble(b) Cost maark with erisp chartes | | | | | |
|---|--|---------|---------|---------|--|--|
| | Ι | II | III | IV | | |
| А | [0,-2] | [-7,-9] | [-3,-5] | [0,-2] | | |
| В | [-4,-6] | [-6,-8] | [-5,-7] | [-4,-6] | | |
| С | [-2,-4] | [-4,-6] | [-3,-5] | [-1,-3] | | |
| D | [-2,-4] | [0,-2] | [-5,-7] | [-2,-4] | | |

| T 1 1 /1 | 0 | | 1.1. | | |
|----------|--------|--------|------|-------|---------|
| I able(b |) Cost | matrix | with | crisp | entries |

Now, using the above table we can apply the Hungarian method to find the assignment for the given problem and the value should be taken from the original table since, it is a maximization problem

Step 4: Select the most negative in the matrix (i.e) is [-7,-9] With this element subtract all the Elements in the matrix. MinZ = -(-MaxZ), the resulting is minimization table

| | 1 | 2 | 3 | 4 | |
|----------|--------|---------|--------------|---------------|------|
| А | [7,7] | [0,0] | [4,4] | [7,7] | |
| В | [3,3] | [1,1] | [2,2] | [3,3] | |
| С | [5,5] | [3,3] | [4,4] | [6,6] | |
| D | [5,5] | [7,7] | [2,2] | [5,5] | |
| luo of C | - [0 2 | 1 [7 0] | 1 - [0 + 7]' | 2 + 0 = 7 - 7 | in t |

Find the value of $C_{11} = [0,-2] - [-7,-9] = [0+7,-2+9] = [7,7]$, in the same it is continued for the remaining cells

Step 5. Iterate towards an Optimal Solution. We proceed according to the Hungarian algorithm and we get optimal solution

| [| 5,5] | [0 | ,0] | [4 | 1,4] | [5, | 5] | |
|---|------|----|-----|----|------|-----|----|--|
| [| 0,0] | [0 | ,0] | [1 | ,1] | [0, | 0] | |
| [| 0,0] | [0 | ,0] | [1 | 1,1] | [1, | 1] | |
| [| 1,1] | [5 | ,5] | [|),0] | [1, | 1] | |
| | | | | | | | | |

We are applying the proposed interval Hungarian method and solve this problem. We get an maximum assignment cost is [18,26] and optimal assignment as A,B,C,D machines are assign to II,IV, I,III operators respectively.

Example 3(Minimize problem):

An air-line operates seven days a week has time-table shown below. Crews must have a Minimum layover (rest) time of 5 hrs, between flights. Obtain the pair of flights that minimizes layover time away from home. For any given pair the crews will e based at the city that result in the smaller layover.

| Delhi-Jaipur | | | Jaipur- Delhi | | | |
|--------------|--------|--------|---------------|-----------|--------|--|
| Flight No | Depart | Arrive | Flight No | Depart | Arrive | |
| 1 | 7.00Am | 8.00AM | 101 | 8.00AM | 9.15AM | |
| 2 | 8.00AM | 9.00AM | 102 | 8.30AM | 9.45AM | |
| 3 | 1.30PM | 2.30PM | 103 | 12.00Noon | 1.15PM | |
| 4 | 6.30PM | 7.30PM | 104 | 5.30PM | 6.45PM | |

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For each pair, mention the town where the crews should be based. **Solution:**

Step1: construct the table for layour times between flights when crew is based at Delhi, for simplicity, consider 15 minutes = 1unit.

| Flight No | 101 | 102 | 103 | 104 |
|-----------|-----|-----|-----|-----|
| 1 | 96 | 98 | 112 | 38 |
| 2 | 92 | 94 | 108 | 34 |
| 3 | 70 | 72 | 86 | 108 |
| 4 | 50 | 52 | 66 | 88 |

Table 1: layover times when crew based at Delhi

Since, the crew must have a minimum layover of 5 hrs between flights

The layover time between flights 1 and 101 will be 24 hrs (96 units)from 8.00 AM to 8.00 AM next day i.e flight 1 arrives jaipur at 8.00 am and leaves the jaipur 8.00 am next day because of minimum layover is 5 hrs between flights and other flights is there in between so flight will be there next day only.

Flight 1 to 102 will be (98units) 8.00 am arrives jaipur leaves jaipur 8.30 am next day= 24 hrs+30 minutes

Flight 1 to 103 will be (112 units) 8.00 am arrives jaipur leaves jaipur 12.00 noon next day= 24 hrs +4 hrs =112 units

Flight 1 to 104 will be (38 units)8.00 am arrives jaipur leaves jaipur 5.30 pm on the same day = 9 hrs + 30 min = 38 mins

The layover time between Flight 2 to 101 will be (9.00 am arrival and depart from jaipur 8.00 am next day) = 23 hrs = 92 units

Flight 2 to 102 will be (9.00 am arrives jaipur and depart from jaipur 8.30 am next day) = 23 hrs +30 minutes = 94 units

Flight 2 to 103 will be (9.00 am arrives jaipur and depart from jaipur 12.00 noon next day) = 24 hrs +3 hrs = 108 units

Flight 2 to 104 will be (9.00 am arrives jaipur and depart from jaipur 5.30 pm same day) = 8 hrs +30 minutes = 34 units

The layover time between Flight 3 to 101 will be (2.30 pm arrival and depart from jaipur 8.00 am next day) =17 hrs + 30 minutes = 70 units

Flight 3 to 102 will be (2.30 pm arrives jaipur and depart from jaipur 8.30 am next day) = 18 hrs = 72 units

Flight 3 to 103 will be (2.30 pm arrives jaipur and depart from jaipur 12.00 noon next day) = 21hrs + 30 minutes = 86 units

Flight 3 to 104 will be (2.30 pm arrives jaipur and depart from jaipur 5.30 pm next day) = 24hrs+3hrs= 108 units

The layover time between Flight 4 to 101 will be (7.30 pm arrival and depart from jaipur 8.00 am next day) = 12 hrs +30 minutes = 50 units

Flight 4 to 102 will be (7.30 pm arrives jaipur and depart from jaipur 8.30 am next day) = 13 hrs = 52 units

Flight 4 to 103 will be (7.30 pm arrives jaipur and depart from jaipur 12.00 noon next day) = 16hrs + 30 minutes = 66 units

Flight 4 to 104 will be (7.30 pm arrives jaipur and depart from jaipur 5.30 pm next day) = 22hrs= 88 units

Step2: Table 2: layover times when crew based at jaipur

| Flight No | 101 | 102 | 103 | 104 |
|-----------|-----|-----|-----|-----|
| 1 | 87 | 85 | 71 | 49 |
| 2 | 91 | 89 | 75 | 53 |
| 3 | 113 | 111 | 97 | 75 |
| 4 | 37 | 35 | 21 | 95 |

Since, the crew must have a minimum layover of 5 hrs between flights

The layover time between flights 101 and 1 will be 21 hrs+ 45 minutes (87 units) from 9.15 AM to 7.00 AM next day by flight no 1 i.e flight 101 arrives Delhi at 9.15 am and leaves the Delhi 7.00 am next day by flight no

1 because of minimum layover is 5 hrs between flights and no other flights is there in between so flight will there next day only.

Flight 101 to 2 will be (91 units) 9.15 am arrives Delhi leaves Delhi 8.00 am next day= 22 hrs+45minutes Flight 101 to 3 will be (113 units) 9.15 am arrives Delhi leaves Delhi 1.30 pm next day= 28 hrs +15 minutes = 113 units

Flight 101 to 4 will be (38 units) 9.15 am arrives Delhi leaves Delhi 6.30 pm on the same day = 9 hrs +15 min = 37 mins

The layover time between Flight 102 to 1 will be (9.45 am arrival and depart from Delhi 7.00 am next day) = 21 hrs+15 minutes = 85 units

Flight 102 to 2 will be (9.45 am arrives Delhi and depart from Delhi 8.00 am next day) = 22 hrs + 15 minutes = 89 units

Flight 102 to 3 will be (9.45 am arrives Delhi and depart from Delhi 1.30 pm next day) = 27 hrs +45 minutes = 111 units

Flight 102 to 4 will be (9.45 am arrives Delhi and depart from Delhi 6.30 pm same day) = 8 hrs +45 minutes = 35 units

The layover time between Flight 103 to 1 will be (1.15 pm arrival and depart from Delhi 7.00 am next day) = 17 hrs + 45 minutes = 71 units

Flight 103 to 2 will be (1.15 pm arrives Delhi and depart from Delhi 8.00 am next day) = 18 hrs + 45 minutes = 75 units

Flight 103 to 3 will be (1.15 pm arrives Delhi and depart from Delhi 1.30 pm next day) = 24 hrs +15 minutes = 97 units

Flight 103 to 4 will be (1.15 pm arrives Delhi and depart from Delhi 6.30 pm same day) = 5 hrs +15 minutes = 21 units

The layover time between Flight 104 to 1 will be (6.45 pm arrival and depart from Delhi 7.00 am next day) = 12 hrs + 15 minutes = 49 units

Flight 104 to 2 will be (6.45 pm arrives Delhi and depart from Delhi 8.00 am next day) = 13 hrs + 15 minutes = 53 units

Flight 104 to 3 will be (6.45 pm arrives Delhi and depart from Delhi 1.30 pm next day) =18 hrs +45 minutes = 75 units

Flight 104 to 4 will be (6.45 pm arrives Delhi and depart from Delhi 6.30 pm same day) = 23 hrs + 45 minutes= 95 units

Step 3: construct the table for minimum layover times between flights with the help of Table 1 and Table 2 layover times denote that the crew is based at jaipur.

| Table 3 | | | | | | |
|-----------|-----|-----|-----|-----|--|--|
| Flight No | 101 | 102 | 103 | 104 | | |
| 1 | 87 | 85 | 71 | 38 | | |
| 2 | 91 | 89 | 75 | 34 | | |
| 3 | 70 | 72 | 86 | 75 | | |
| 4 | 37 | 35 | 21 | 88 | | |

Step 4:

We take a linear assignment problem as an example problem and solved this problem by traditional Hungarian method .The assignment cost of assigning any operator to any one machine is given in the following table

Cost matrix with crisp entries

| Flight No | 101 | 102 | 103 | 104 |
|-----------|---------|---------|---------|---------|
| 1 | [86,88] | [84,86] | [70,72] | [37,39] |
| 2 | [90,92] | [88,90] | [74,76] | [33,35] |
| 3 | [69,71] | [71,73] | [85,87] | [74,76] |
| 4 | [36,38] | [34,36] | [20,22] | [87,89] |

Now, using the above table we can apply the Hungarian method to find the assignment for the given problem and the value should be taken from the original table since, it is a minimization problem

Step 5. Iterate towards an Optimal Solution. We proceed according to the Hungarian algorithm and we get optimal solution

| Flight No | 101 | 102 | 103 | 104 |
|-----------|---------|-------|--------------------|-----------|
| 1 | [4,4] | [0,0] | [0,0] | [0,0] |
| 2 | [12,12] | [8,8] | [8,8] | [0,0] |
| 3 | [0,0] | [0,0] | [28,28] | [50,50] |
| 4 | [4,4] | [0,0] | [0,0] | [100,100] |

The optimal assignments are

Flight 1-102 Flight 2-104 Flight 3-101

Flight 4-103

Example 4(Alternate optimal solution):

An automobile workshop wishes to put four mechanics to four different jobs. The mechanics have somewhat different kinds of skills and they exhibit different levels of efficiency from one job to another. The manager of the workshop has estimate the number of man-hours that would be required for each job-man combination. This is given in the matrix form in adjacent table

| Job Mechanic | А | В | С | D |
|-----------------|---|---|---|---|
| 1 | 5 | 3 | 2 | 8 |
| 2 | 7 | 9 | 2 | 6 |
| 3 | 6 | 4 | 5 | 7 |
| 4 | 5 | 7 | 7 | 8 |

Find the optimum assignment that will result in minimum man -hours needed

Step 1:

We take a linear assignment problem as an example problem and solved this problem by traditional Hungarian method .The assignment cost of assigning any operator to any one machine is given in the following table

Cost matrix with crisp entries

| | А | В | С | D |
|---|-------|--------|-------|-------|
| 1 | [4,6] | [2,4] | [1,3] | [7,9] |
| 2 | [6,8] | [8,10] | [1,3] | [5,7] |
| 3 | [5,7] | [3,5] | [4,6] | [6,8] |
| 4 | [4,6] | [6,8] | [6,8] | [7,9] |

Now, using the above table we can apply the Hungarian method to find the assignment for the given problem and the value should be taken from the original table since, it is a minimum problem

Step 2: We make the 'zero-assignments' as shown in the table .It may be note that an assignment problem can have more than one optimal solution .the other solution is shown in table

| Job Mechanic | A | | В | | С | , | D | |
|-----------------|----|------|----|-----|----|-----|----|------|
| 1 | [2 | 2,2] | [0 | ,0] | (j | ્રા | [2 | 2,2] |
| 2 | [4 | 1,4] | [6 | ,6] | [0 | ,0] | ¥ | LØJ |
| 3 | [2 | ,2] | [0 | ,0] | [2 | ,2] | [(| ,0] |
| 4 | [(|),0] | [2 | ,2] | [3 | ,3] | [(|),0] |

| Optimal solution I | | | | | | | |
|--------------------|-----|------------|--|--|--|--|--|
| Mechanic | Job | Man -Hours | | | | | |
| 1 | В | [2,4] | | | | | |
| 2 | С | [1,3] | | | | | |
| 3 | D | [6,8] | | | | | |
| 4 | А | [4,6] | | | | | |

| | | 1 | | | | | | |
|----------|---------------------|-------|-----|-------|------------|-----|---|-------|
| | Job | Α | В | | C | | D | |
| Mechanic | | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| | 1 | [2,2] | D | Q1 | [0 | ,0] | | [2,2] |
| | 2 | [4,4] | [6 | 6] | [0 | ,0] | | [0,0] |
| | 3 | [2,2] | [0] | 0] | [2 | ,2] | | [0,0] |
| | 4 | [0,0] | [2 | 2] | [3 | ,3] | | [0,0] |
| | | | | | | | | |
| | Optimal solution II | | | | | | | |
| | : | J | Job | | Man -Hours | | | |
| | С | | | | [1,2] | | | |
| | | | D | | [5,7] | | | |
| | | | | [3,5] | | | | |
| | | А | | | [4,6] | | | |
| | | | | | | | | |

VI. CONCLUSIONS

In this paper, a new and simple modal was introduced for solving assignment problems. As considerable number of problems has been so for presented for Assignment problem in which the Hungarian method is more convenient method therefore this paper present a three different models for solving assignment problems, the proposed interval Hungarian method is effective and useful in this interval context. Using this method we can solve real world linear assignment problems where entries of the cost matrix are interval form. Generalized linear assignment problems can be solved by this proposed method.

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